

The University of Chicago

THE ROLE OF MONEY IN EQUILIBRIUM
CAPITAL THEORY

A PART OF A DISSERTATION SUBMITTED TO THE FACULTY
OF THE DIVISION OF THE SOCIAL SCIENCES IN CANDIDACY
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF ECONOMICS

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By
MARTIN BRONFENBRENNER

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INTRODUCTION

WITH THE STUDY OF problems of capitalization, credit, and discount, the peaceful parade of equationally determinate barter economy¹ comes to an abrupt halt. The systems of equations expressing the conditions of equilibrium capital theory differ in number from the unknowns to be determined. Equational determinacy is established only by the introduction of money into the system. This conclusion is, I believe, novel in general-equilibrium theory. It is an obvious confirmation to an *essentially* monetary theory of interest—as distinguished from a “real capital” theory admitting of short-run monetary dislocations. In its support, demonstration of the following subordinate propositions will be attempted:

In a static, but not stationary,² barter economy it is necessary, first, to posit a plurality of interest rates in order to secure the equality of demand and supply of loans for each of the various goods and services lent *in natura*.³ It is necessary, second, to provide for yet another plurality of discount rates, to take account of the loan aspect of payments for services made at times other than those at which their final products are sold. It is necessary, third, to establish various fixed relationships, which I call capitalization rates, between the prices of capital services and the prices of the capital goods which are their sources, so that individuals may be able to choose rationally between the purchase (sale) of the goods and the purchase (sale) of the services. Under barter, there are no clear interrelationships between these various rates. The lack of interrelationships renders indeterminate each of the three

¹ A barter economy is defined here as one in which no commodity is sufficiently liquid to all individuals to have general cash-balance use. Use of any existing commodity as *numéraire*, or measure of value, is quite consistent with barter, under this definition. (It is, as Walras has shown, necessary to the achievement of equilibrium in any multi-commodity market.) The *numéraire*, whatever it may be, does not acquire liquidity or other properties increasing its utility as a result of its *numéraire* status. It is treated as an ordinary consumption good.

² A static economy is defined here as one in which the number of economic subjects, their tastes as represented by utility functions, and their techniques as represented by production functions, remain fixed over time. A stationary economy, on the other hand, satisfies two additional conditions: the prices of all goods and services, and the quantity of each type of capital instrument in existence, remain fixed as well.

³ The plurality of interest and discount rates is eliminated, but without the achievement of equilibrium, in the special stationary case in which prices are constant through time.

primary branches of the theory—capitalization, credit, and discount. The introduction of money eliminates these indeterminacies, provided that certain economically reasonable restrictions are placed even on the money economy.

CAPITALIZATION UNDER BARTER

The Unknowns.—In capitalization theory proper five sets of economic unknowns are to be determined: First, the quantities of C different capital goods (a) produced and (b) held by each of θ individuals. The number of these unknowns is $2C\theta$. Second, the C prices of the various capital goods. Third, the production allotments or coefficients (quantities of productive services used by each individual) in the production of each type of capital good. If there are c capitalizable services and d noncapitalizable services (types of labor), the total number of production allotments to be determined is $(c+d)C\theta$. Fourth and last, the profits derived by each individual from the production of each capital good. There are clearly $C\theta$ of these. The total number of unknowns introduced up to this point is $(c+d+3)C\theta+C$.⁴

The Expansions.—Let there be presupposed the existence and stability of general equilibrium with regard to capitalless exchange and production under barter. The conventional equation systems of the Lausanne School may be expanded to allow the introduction of capital goods.

⁴ Not included in this preliminary list of unknowns are certain important mathematical entities which will enter in the course of later discussion. These are the capitalization rates already mentioned. They are omitted because their number is itself a subject for special consideration.

Also omitted from the list of unknowns are the lengths of life and rates of depreciation of capital goods. These are not determined by the decisions of any single period, since the decisions of any moment are limited by those of the past and when adopted are only tentative and subject to continuous revision. In a nonstationary economy, there need exist no "consistent" depreciation policies over time. Frankly literary treatment of these topics therefore appears more realistic as well as simpler than much of the recent mathematical development. See, however, for a survey of the mathematical literature and for independent advances, G. A. D. Preinreich, "Annual Survey of Economic Theory: The Theory of Depreciation," *ECONOMETRICA*, Vol. 6, July, 1938, pp. 219-241; "The Practice of Depreciation," *ibid.*, Vol. 7, July, 1939, pp. 235-265; "The Economic Life of Industrial Equipment," *ibid.*, Vol. 8, January, 1940, pp. 12-44 (appendix includes comments by Harold Hotelling); and "Note on the Theory of Depreciation," *ibid.*, Vol. 9, January, 1941, pp. 80-88; also K.-G. Hagstroem, "Remarks on the Theory of Depreciation," *ibid.*, Vol. 7, October, 1939, pp. 289-303.

A final omission from the list of unknowns is any "period," whether of production, investment, or turnover. This omission is for similar reasons, but will be accorded separate discussion below.

The general-equilibrium theory of exchange includes three systems of equations, expressing respectively:

(I) The principle of rationality in consumption. Each individual, maximizing utility with a fixed income, apportions expenditures in such wise that the marginal utility of a penny's worth of each good is equal for him. (Or, alternatively, so that the ratio of the prices of any two goods equals their marginal rate of substitution.)⁵

(II). The principle of invariance of quantities. The quantity of each good remains constant throughout the process of exchange.⁶

(III) The principle of the invariance of values. Each individual's assets, measured in terms of the *numéraire*, remain fixed throughout the exchange process.⁷

The numbers of equations in Systems (I) and (III) remain unchanged when capital goods are introduced formally into the economy. Capital goods are not considered to have “independent” marginal utility, hence System (I) is unchanged. Individual equations of (III) change their form to allow for a wider choice in purchasing, but their number remains the same. C new equations are added to (II).

⁴ Following Pareto, we can denote the utility function by ϕ and prices by p . If the *numéraire* is a and other consumption goods b, b', b'' , individuals are $1, 2, \dots, \theta$, and the marginal utility of income or assets is μ , the mathematical expression of (I) is the well-known

$$\begin{aligned} \mu_1 &= \phi_{a1} = \frac{\phi_{b1}}{p_b} = \frac{\phi_{b'1}}{p_{b'}} = \frac{\phi_{b''1}}{p_{b''}} = \dots, \\ \mu_\theta &= \phi_{a\theta} = \frac{\phi_{b\theta}}{p_b} = \frac{\phi_{b'\theta}}{p_{b'}} = \frac{\phi_{b''\theta}}{p_{b''}} = \dots. \end{aligned}$$

* If we denote the quantities of goods held by consumers after exchange by q , with appropriate commodity subscript, and the quantities held prior to exchange by q^0 , we can express (II) by:

$$\begin{aligned} \sum_{i=1}^{\theta} q_{a_i}^0 &= \sum_{i=1}^{\theta} q_{a_i}, \\ \sum_{i=1}^{\theta} q_{b_i}^0 &= \sum_{i=1}^{\theta} q_{b_i}, \\ &\vdots \end{aligned}$$

⁷ Or, symbolically, in the terminology already developed:

[illegible]

Systems (II) and (III) are not completely independent. The sum of (III) is equal to the sum of (II), when each member of (II) is multiplied by the appropriate price. One equation of (II) or (III) is accordingly omitted from the number of independent equations in the Walrasian or Paretian system.

The conventional general-equilibrium theory of production likewise includes three equation systems, expressing respectively:

(IV) The given technical relations between the quantity of any good produced and the production allotments in its production. These are known as production functions.⁸

(V) The principle of efficiency in production. Each producer maximizes his output from a given entrepreneurial budget so that the anticipated marginal productivities (efficiencies) of additional increments of all productive services are equal.⁹ (I abstract from "limitational" productive services, and the problems they raise.)

(VI) The principle of rationality in production. Each producer will carry production of each commodity to the point where the incremental cost of additional output is equal to the additional revenue derived from it.¹⁰ Under pure or monopolistic competition, each equation of this group may be replaced by an equation of total costs and total receipts, if the equilibrium is discussed as a long-run one.

These Systems expand in a purely mechanical manner when capital

⁸ These production functions are denoted by f ; capitalizable productive services are $c, c', c'', \dots$; noncapitalizable productive services are $d, d', d'', \dots$; the production allotment of service c , for example, in producing commodity b is c_b . With these symbols, (IV) becomes:

$$q_{a1}^0 = f_{a1}(c_{a1}, \dots, d_{a1}, \dots), \dots, q_{b1}^0 = f_{b1}(c_{b1}, \dots, d_{b1}, \dots), \dots, \\ \dots, \dots, \\ q_{a\theta}^0 = f_{a\theta}(c_{a\theta}, \dots, d_{a\theta}, \dots), \dots, q_{b\theta}^0 = f_{b\theta}(c_{b\theta}, \dots, d_{b\theta}, \dots) \dots$$

⁹ System (V), derived in part from (IV) is, in this terminology:

$$p_c = \frac{\partial f_{a1}}{\partial c_{a1}} = \frac{\partial f_{b1} p_b}{\partial c_{b1}} = \dots = \frac{\partial f_{a\theta}}{\partial c_{a\theta}} = \frac{\partial f_{b\theta} p_b}{\partial c_{b\theta}} = \dots, \\ \dots, \dots, \\ p_d = \frac{\partial f_{a1}}{\partial d_{a1}} = \frac{\partial f_{b1} p_b}{\partial d_{b1}} = \dots = \frac{\partial f_{a\theta}}{\partial d_{a\theta}} = \frac{\partial f_{b\theta} p_b}{\partial d_{b\theta}} = \dots, \\ \dots, \dots, \dots$$

In this form, (V) allows for monopolistic but not for monopsonistic imperfections in commodity and productive-service markets. It can be modified simply to cover the monopsony case.

¹⁰ In the present notation, the conditions of maximum profit may be written:

$$1 = \frac{d(c_{a1} p_c + \dots + d_{a1} p_d + \dots)}{dq_{a1}^0} = \dots = \frac{d(c_{a\theta} p_c + \dots + d_{a\theta} p_d + \dots)}{dq_{a\theta}^0}, \\ \frac{d(q_{b1}^0 p_b)}{dq_{b1}^0} = \frac{d(c_{b1} p_c + \dots + d_{b1} p_d + \dots)}{dq_{b1}^0}, \\ \dots, \dots, \\ \frac{d(q_{b\theta}^0 p_b)}{dq_{b\theta}^0} = \frac{d(c_{b\theta} p_c + \dots + d_{b\theta} p_d + \dots)}{dq_{b\theta}^0}, \\ \dots, \dots, \dots$$

goods are added to the products under consideration.¹¹ System (IV) provides $C\theta$ more production functions; System (V) provides $(c+d)C\theta$ more production allotments; System (VI) provides $C\theta$ more equations of marginal revenue and cost.

The results of the preceding expansions are summarized in Table I. As a final conclusion, there apparently remain only $C\theta$ unknowns to be determined by special equation systems of capitalization theory for the necessary condition of an equilibrium solution to be fulfilled.

TABLE I
EXPANSION OF SYSTEMS OF EXCHANGE AND OF PRODUCTION WHEN THE
MARKET IS EXTENDED TO CAPITAL GOODS

Added Unknowns	Number	Added Equations	Number
Quantities of Capital Goods Held	$C\theta$	System (II)	C
Quantities of Capital Goods Produced	$C\theta$	System (IV)	$C\theta$
Prices of Capital Goods	C	System (V)	$(c+d)C\theta$
Production Allotments in Capital Goods Production	$(c+d)C\theta$	System (VI)	$C\theta$
Profits in Capital Goods Production	$C\theta$		
Total	$(c+d+3)C\theta+C$	Total	$(c+d+2)C\theta+C$

Systems of Capitalization Proper.—The foregoing expansions have provided no connection between the quantities of capital goods possessed and the quantities of capitalizable services dealt in. Neither have they provided any connection between the prices of the two classes of commodities. An obvious task for a distinct and separate capitalization theory is the specification of these connections.

Suppose that an individual has determined to exchange certain quantities of capital goods in order to obtain other goods and services. He may sell the capital goods outright and perform the exchange in that manner. On the other hand, he may sell the services of the capital goods, an operation which may involve renting the goods themselves.

¹¹ Difficulties arise throughout these expansions whenever certain commodities may be regarded simultaneously as capital goods—viz., coal, source of the service of heat—and as capitalizable services—viz., coal, product of a coal mine. The expansions indicated are then illegitimate. In production theory generally, however, it will be found that for each illegitimate expansion there is one less unknown as well as one less equation. In the theory of exchange the illegitimate expansions may be replaced by equations stating the proportionality or equality of price between the commodity considered as capital good and the same commodity considered as a capitalizable service.

(Another alternative, utilizing the capital goods, or rather their services, for direct production, is considered in the theory of production.) He must decide rationally which of the above alternatives he shall take with his holdings of particular capital goods, or in what proportions he shall take both. We choose to represent him as maximizing his income from capitalizable services, constrained by the fixation by production or prior exchange of his stock of capital goods. The mathematics of a rational choice can be stated in terms of a Lagrange multiplier:¹²

$$(1) \quad q_c(p_c - kP_c) + q_{c'}(p_{c'} - k'P_{c'}) + q_{c''}(p_{c''} - k''P_{c''}) + \dots - \lambda(Q_c P_c + Q_{c'} P_{c'} + Q_{c''} P_{c''} + \dots) = \max. \quad (\theta \text{ equations})$$

Here the capital letters $P, Q, C, \dots$ refer to capital goods, and the corresponding lower-case letters to their capitalizable services per unit time. $C, C', C'', \dots$ are different capital goods, and $c, c', c'', \dots$ are their respective capitalizable services. The k 's of equation (1) represent the anticipated cost per time period to the owner of the capital good of obtaining the capitalizable service from such good. They are expressed as fractions of the price of the capital good, under the assumption that each k will be approximately the same, whether individuals use capital goods themselves or rent them to others. These terms are variable as between individuals for the same capital good, as well as between goods for the same individual. They are determined by anticipations of price changes as well as by purely technical considerations, and are not the arbitrary constants which Walrasian capital theory assumes them to be.¹³ Their importance in the present theoretical development is as determiners of individuals' willingness to capitalize at market prices.

According to equations (1), each individual wishes to maximize his return from the sale of capital services, but is so constrained that he cannot change the total value of his holdings of their sources by deciding either to sell or not to sell them. Differentiating with respect to $q_c, q_{c'}, \dots$, we obtain:

$$(2) \quad \begin{aligned} (p_c - kP_c) + q_c \frac{\partial(p_c - kP_c)}{\partial q_c} &= \lambda \left(P_c \frac{\partial Q_c}{\partial q_c} + Q_c \frac{\partial P_c}{\partial q_c} \right), \\ (p_{c'} - k'P_{c'}) + q_{c'} \frac{\partial(p_{c'} - k'P_{c'})}{\partial q_{c'}} &= \lambda \left(P_{c'} \frac{\partial Q_{c'}}{\partial q_{c'}} + Q_{c'} \frac{\partial P_{c'}}{\partial q_{c'}} \right), \\ &\dots \end{aligned} \quad (C\theta \text{ equations})$$

¹² Similar equations arise if the choice is between the *purchase* of capital goods and the *purchase* of their services. The constraint then takes the form of the total "budget" to be spent on such goods or services.

¹³ Compare Léon Walras, *Eléments d'économie politique pure*, édition définitive (Paris, 1926), esp. p. 243.

No individual can alter finitely the price of either capital good as compared to that of its service by choosing, at the margin, to buy or sell either the one or the other. The terms involving relative price changes therefore drop out of equations (2), even under conditions of imperfect competition,

Equations (2) may be simplified still further, since $(\partial Q/\partial q)$ may be shown equal to unity. This appears fallacious dimensionally; are we equating a coal mine to a ton of coal? No, for the service of a coal mine is being measured in terms of its total output per period, not in tons of coal per period. The dimensionality of q is then precisely QT^{-1} , so that the dimensionality of $(\partial Q/\partial q)$ is (Q/QT^{-1}) , or T . Over a unit period of time, $(\partial Q/\partial q)$ has the value of unity.¹⁴

In their final simplification, equations (2) become the new System of capitalization theory, numbered (VII):

$$\begin{aligned} \frac{p_c - k_1 P_C}{P_C} &= \frac{p_{c'} - k_1' P_{C'}}{P_{C'}} = \dots = \lambda_1, \\ \text{(VII)} \quad &\dots \dots \dots \text{(C}\theta \text{ equations)} \\ \frac{p_c - k_\theta P_C}{P_C} &= \frac{p_{c'} - k_\theta' P_{C'}}{P_{C'}} = \dots = \lambda_\theta. \end{aligned}$$

This System introduces $C\theta$ new equations,¹⁵ this being the number apparently required to equate the number of equations to the number of unknowns in Table I, and to provide the necessary condition for an

¹⁴ If C is a coal mine producing 1000 tons a week, and c is its service, then c is measured in units of 1000 tons per week, and $(\partial Q/\partial q)$ is clearly unity. A single ton of coal is 1/1000 of a unit of c . The problem is more complex when a capital good gives simultaneous services of different kinds. If C' is a well producing both oil and natural gas, we should consider the relative prices of the oil and gas to determine the proportion of each unit of c' represented by a barrel of oil or a cubic foot of gas. Professor Jacob Marschak would solve this problem by writing, for each individual, a single production function involving all assets and all productive services at once; "Money and the Theory of Assets," *ECONOMETRICA*, Vol. 6, October, 1938, p. 313.

¹⁵ There is no conflict here with the more accurate presentation which regards the price of the capital good as the capitalized value of the entire continuous future net income stream derivable from the use of the good, with proper discounting for futurity. The two presentations are alternative. My own preference for Walras' method is based on the greater simplicity of his notation. The concepts of continuous stream and discounting for futurity are unfortunately buried by the Walrasian presentation in terms corresponding to our k 's (his μ 's and ν 's). These should be interpreted as allowances for depreciation and insurance sufficient to provide for perpetuation of the instrument's productivity, and likewise for the futurity of the yields of subsequent periods. There is no reason to believe that Walras himself utilized capital concepts not fully developed until after his death.

equilibrium on the capital-goods market. The solution by Lagrange multipliers has, however, introduced θ new mathematical unknowns per period, or $n\theta$ in all, and the solution is not complete without offsetting equations. Their economic meaning should be clear immediately from (VII). Each is a capitalization rate—a “marginal efficiency” expressed as a percentage of purchase price. By (VII) this rate is uniform for any individual as between different capital goods C , C' , C'' , $\dots$. A second proposition relating to this rate is usually assumed without proof, namely, that it is uniform as between individuals also. This need not be true, however, in the absence of a loan market, except when k for some particular good is constant as between individuals. Only in this case may λ be expressed as a quotient of quantities which do not vary as between individuals.

In a loanless barter economy, capitalization rates which vary from person to person are not impossible, as they may appear at first glance. An individual capitalizing at a relatively low rate may be willing to pay (or accept) the market prices for both capital goods and their services. He may simply write down his capital goods at a faster rate (with a larger k) than does someone else whose capitalization rate is higher and who likewise pays (or accepts) market prices for both capital goods and their services.

The system is therefore indeterminate; its equations number θ less than its unknowns. It is not in general rendered determinate by a substitution of stationary for static equilibrium.¹⁶ The additional conditions necessary to render the system stationary are C in number, equating the value of capital produced to the amount written off the economy's books by depreciation, etc. Needing θ more equations to set up a determinate theory of capitalization, we are provided C equations of type (3) below, expressing a “quantity-of-capital” criterion of stationariness. (Q'_C , $Q'_{C'}$, $\dots$ represent quantities of capital goods C , C' , $\dots$ produced.) Equilibrium is reached only in the meaningless special case in which the number of individuals and the number of capital goods are equal.

$$\begin{aligned}
 (3) \quad & \sum_{i=1}^{\theta} k_i Q_{Ci} = \sum_{i=1}^{\theta} Q'_{Ci} \\
 & \sum_{i=1}^{\theta} k'_i Q_{C'i} = \sum_{i=1}^{\theta} Q'_{C'i}, \\
 & \dots \dots \dots
 \end{aligned}
 \qquad (C \text{ equations})$$

At this point, a characteristic aspect of the so-called Austrian theory

¹⁶ For the distinction between static and stationary conditions, *vide supra*, note 2.

of capital beckons strongly. In accord with the position of Böhm-Bawerk and his successors, we might postulate an inverse relationship between each individual's capitalization rate and his anticipated and given investment, production, or turnover "period." We may use as a measure of the latter either the reciprocal of k , as suggested by Mr. Nicholas Kaldor, or the weighted average of specific subsidiary investments derived by numerous writers from the "equation of accounts" of financial mathematics and by Professor J. R. Hicks from the elasticity of capital value with respect to the discount ratio.¹⁷

Such a system of equations, unfortunately, is of doubtful legitimacy in a barter economy. Paradoxical as the statement may seem, the investment period is a monetary phenomenon in so far as it has economic significance at all. It has causative significance, that is to say, only in a monetary economy. It is a measure of the time which is expected to elapse before liquid funds, frozen in some investment plan, will be returned in liquid form for consumption or reinvestment without recourse to a forced sale of assets. The notion of a single commodity pre-eminent in liquidity, conversion from and to which mark the beginning and the end of the period, underlies the entire development. In a barter economy, even with uncertainty present, differentials of liquidity between commodities are by definition slight and unimportant, and the entire economic motivation for preferring shorter to longer investment periods is absent.

CREDIT UNDER BARTER

Introduction.—The primary reason for turning to the theory of credit to obtain equilibrating conditions for the theory of capitalization is to be found in the close relationship existing between the two subjects. When individuals wish to alter the distribution of their incomes over time, they may do so by obtaining capital goods and renting them or selling their services. This possibility is considered in the theory of capitalization. On the other hand, they may lend existing (or newly-produced) stocks of goods at interest, capital as well as consumption goods. The markets for capital goods and for loans are accordingly related intimately, as are the theories of price formation on these markets.

The Unknowns.—If attention is confined to a single point in time, or exchange interval, turning to the loan market necessitates the inclusion

¹⁷ Kaldor's definition is suggested in the course of his "Annual Survey of Economic Theory: The Recent Controversy on the Theory of Capital," *ECONOMETRICA*, Vol. 5, July, 1937, p. 213, as one suggested method of rehabilitating the production-period concept, suffering under the attacks particularly of Professor F. H. Knight. Hicks's definition is to be found on p. 186 of *Value and Capital*, Oxford, 1939.

in the general-equilibrium system of $G(\theta+1)$ new unknowns.¹⁸ Of these, $G\theta$ represent the quantities of each good which each individual will lend or borrow. The remaining G unknowns are the prices, or interest charges, for the lending of each good or service over the exchange interval.¹⁹ They are indicated by $i_a, i_b, \dots, i_c, \dots, i_c, \dots$.

The Equations.—The greater part of the formal theory of credit may be obtained by expansions of Systems (I–II) of the theory of exchange. An independent System is required to eliminate arbitrage between commodities, and this System has the effect of rendering the theory of credit overdeterminate.

System (I), it will be remembered, gives mathematical expression to the so-called principle of rationality in consumption. At any time, the marginal utility of a good for future disposal is different from (usually, though probably not always, less than) its marginal utility for present disposal. In determining how much of a certain stock the holders wish to lend or to borrow, the economist visualizes the economic subjects as weighing the presumably greater marginal utility of goods for present disposal as against the future benefits of presumably increased income.

We have indicated by ϕ and μ the utility function of present goods and the marginal utility of present income, respectively. Let us further denote by ϕ' and μ' the utility function of future goods and the marginal utility of future income. Let us introduce further the symbol A_b , equal to unity plus the rate of interest per period on any good b . The present value of b is (p_b/A_b) . Under conditions of equilibrium between present and future goods, we must have, for individuals $1, \dots, \theta$:

$$\begin{aligned}\mu_1 &= \mu'_1, \dots, \mu_\theta = \mu'_\theta, \\ \phi_{b1} &= \frac{\phi_{b1}'}{p_b/A_b}, \dots, \phi_{b\theta} = \frac{\phi_{b\theta}'}{p_b/A_b}, \\ \phi_{b'1} &= \frac{\phi_{b'1}'}{p_{b'}/A_{b'}}, \dots, \phi_{b'\theta} = \frac{\phi_{b'\theta}'}{p_{b'}/A_{b'}}, \\ &\dots, \dots, \dots\end{aligned}$$

Furthermore, by the same process of constrained maximization of ϕ which led to System (I), we have:

¹⁸ Let the sum total of all goods and services which can be lent or borrowed be denoted by G . The maximum possible value of G is $(1+b+c+d+C)$.

¹⁹ This implies a single interest charge per period on loans of each good in a particular period, regardless of the length of the loan. Some such postulate is necessary to eliminate arbitrage, though as Hicks has shown (*op. cit.*, p. 145), it is not strictly accurate when anticipated changes in the interest rate are considerable. The G interest rates on loans of different commodities are also assumed independent of each other. This last assumption will be modified in the next section of this essay.

$$\begin{aligned}
 \mu_1 = \dots = \mu_1' &= \frac{A_b \phi_{b1}}{p_b} = \frac{A_{b'} \phi_{b'1}}{p_{b'}} = \dots, \\
 (4) \quad &\dots \dots \dots (C\theta \text{ equations}) \\
 \mu_\theta = \dots = \mu_\theta' &= \frac{A_b \phi_{b\theta}}{p_b} = \frac{A_{b'} \phi_{b'\theta}}{p_{b'}} = \dots.
 \end{aligned}$$

Equations (4) represent the familiar "agio" or "time-preference" aspects of the theory of capital. An additional set of G equations may be added, representing the invariance of quantities, indicating that the amount of each commodity lent per period is equal to the amount borrowed. Denoting amounts of $a, b, \dots$ lent or borrowed by $q_a^*, q_b^*, \dots$, which are positive for loans and negative for borrowings, for θ individuals we have, as in (II):

$$\begin{aligned}
 \sum_{i=1}^{\theta} q_{ai}^* &= 0, \\
 (5) \quad \sum_{i=1}^{\theta} q_{bi}^* &= 0, & (G \text{ equations}) \\
 \sum_{i=1}^{\theta} q_{b'i}^* &= 0.
 \end{aligned}$$

The total number of equations (4) and (5), namely $(\theta+1)G$, equals the number of unknowns of the theory of credit. They are not, however, sufficient to eliminate arbitrage by means of simultaneous loans and borrowings of different commodities. To illustrate: lending a capital good for a period is identical both with selling and with lending its services for the same period, in so far as economic effects are concerned. If i_C (the interest rate on capital good C) $\geq \lambda_i$ for individual i , he will prefer lending the good to selling its services (or vice versa, with the lower branch of the inequality).²⁰ If $i_C \geq i_c$, the latter being the rate of interest on loans of the *services* of C , individuals will lend the good rather than the services (or vice versa). It will in fact be profitable to borrow and lend simultaneously on different markets. Opportunity for such arbitrage is inconsistent with an equilibrium situation, and cannot be eliminated by the equation systems adduced up to this point. To eliminate it, we must have as an additional condition for equilibrium:

²⁰ Mr. Keynes carries this argument further, suggesting in his *General Theory of Employment, Interest and Money*, New York and London, 1936, p. 223, that the capitalization rate (marginal efficiency) under barter conditions would equal the *highest* commodity-rate of interest ("own-rate") on the loan market, "because it is the greatest of these rates that the marginal efficiency of a capital asset must attain if it is to be newly produced."

$$(6) \quad i_c = i_{c'} = \dots = i_c = i_{c'} = \dots = \lambda_1 = \lambda_2 = \dots = \lambda_\theta.$$

($c + C + \theta - 1$ equations)

Of these equations, θ serve to balance undetermined variables of the theory of capitalization by bringing about equality of individuals' capitalization rates. The remainder do not have counterbalancing variables. Their introduction leaves the theory of credit in an overdeterminate condition. Introduction of still more equations relating commodity interest rates to each other via futures markets would increase the overdeterminacy.²¹

A Natural Rate of Interest?—Were a single rate of interest on loans of all commodities to be assumed, this rate being necessarily equal to the capitalization rates of all individuals, (6) would be expanded to a set of $(G + \theta - 1)$ equations, and the theory of credit would become further overdeterminate. In the special stationary case, however, such a rate must be presumed to exist. For if equilibrium prices, once arrived at, are supposed constant through time, different rates on different commodities are easily seen to permit arbitrage.²² (Inconsistencies of anticipation prevent this when the equilibrium is a moving one.) Under stationary assumptions, then, equations (6) must be expanded, with added overdeterminacy as a consequence.

We can go further, and visualize a stationary state in which not only (a) the quantity of each type of capital good and (b) each price is constant, but also in which no person has any motive to borrow with a view to future repayment or lend with a view to future consumption. In such a state, there is no theory of loans, merely a theory of capitalization. All capitalization rates must be set equal to each other, as in (6), in order to eliminate arbitrage through the borrowing and lending of *capital* goods and services. This treatment still leaves a single unknown of capitalization theory to be determined, the uniform capitalization rate λ . To solve for λ ,²³ the common device is the construction of an

²¹ The legitimacy of such equations is doubtful, not only because of the mutual inconsistency of various traders' anticipations, but because of the dependence of speculative profits on forecasting the guesses of others rather than the actual course of events. Compare Keynes, *op. cit.*, p. 154 f., and Nicholas Kaldor, "Speculation and Economic Stability," *Review of Economic Studies*, Vol. 7, October, 1939, p. 8.

²² The same conclusion holds when price changes are foreseen perfectly by all economic subjects—though why they do not then occur immediately is not made clear by Professor F. A. von Hayek, who discusses this case in *The Pure Theory of Capital*, London, 1941, p. 67: "Measured in terms of any one commodity, the rate of increase will have to be the same for all commodities. The actual numerical value of this rate of increase will, however, be different according as one commodity or another is chosen as the standard of comparison or 'numéraire'."

²³ This terminology is not strictly accurate. No single set of equations "deter-

equation stating that all sums not spent by individuals on consumption goods for immediate consumption, or on productive services for immediate use, are devoted to the purchase of capital instruments. Such an equation may be found in the works of Walras and Pareto, among general-equilibrium writers.²⁴ This treatment consciously or unconsciously eliminates consumption loans and alteration of stocks or inventories. An apparently determinate barter theory of capitalization and credit is achieved, including a natural rate of interest, but it is a theory of a stationary economy, with the assumption of stationariness usually kept implicit.

Immediately we drop stationary assumptions, economic justification for postulating a single "natural" rate of interest is lost. Mathematical justification was never present. Despite the obvious advantages which interest and monetary theorists have obtained by postulating such a rate, its use is unjustified in a barter economy. Under barter, a single rate of interest, far from being a necessary condition of equilibrium, serves only to throw further out of alignment a credit theory already overdeterminate.²⁵

DISCOUNT UNDER BARTER

For any individual, the rate of discount must bear a simple relationship to the rate of interest which he could obtain by lending part or all of his working capital instead of paying for production which is embodied in products which he can sell only in the future, or buying commercial paper which will be redeemed only in the future. This relation between δ , the rate of discount, and i , the rate of interest, is familiar:

$$(7) \quad (1 + i)(1 - \delta) = 1.$$

If the time interval of discount is short, equalization of interest and discount rates is a legitimate first approximation.

But under barter conditions, equations of type (7) may not be used to solve for the $n\theta$ unknown discount rates which the theory of discount introduces. The rate of interest, i , varies from person to person, depending on the commodity composition of whatever working capital he may

mines" its own special subset of unknowns. All is rather a single process of mutual determination. There is nevertheless a natural tendency for the economist to relate particular equations to particular unknowns by reason of numbers, subject matter, or both.

²⁴ Walras, *op. cit.*, Leçons 23–26; Pareto, *Cours d'économie politique*, Lausanne, 1896, Vol. I, p. 60, and "Le equazioni dell'equilibrio dinamico," *Giornale degli Economisti*, Series II, Vol. 23, 1901, p. 257.

²⁵ Several recent writers have pointed out the plurality of "natural" or "barter" interest rates. See, e.g., Hayek, *op. cit.*, pp. 37, 168 f., 231 f., Keynes, *op. cit.*, p. 222 f., and A. C. Pigou in his review of Keynes's *General Theory*, in *Economica*, Vol. 3 (N.S.), May, 1936, pp. 115–132.

consider lending. (If this consists of capital goods exclusively, we have seen that $i = \lambda$, and as a capitalization rate is also uniform as between individuals.)²⁶ Otherwise, with no unique i , the barter theory of discount introduces no new equations at all, and reduces formally to θ unknowns. It is accordingly indeterminate, except under the special conditions in which a single rate of interest can be applied.

A SUMMARY TABLE

Table II, summarizing the results to this point, indicates an equilibrium solution only in case $\theta + 1 = c + C$. In the general case, the indeterminacy of capitalization and discount fails to cancel the overdeterminacy of credit.

TABLE II
INDETERMINACY OF THE BARTER THEORY OF CAPITALIZATION,
CREDIT, AND DISCOUNT

Unknowns	Number	Equations	Number
Quantities of Capital Goods Held	$C\theta$	System (I)	$G\theta$
Quantities of Capital Goods Produced	$C\theta$	System (II)	$C + G$
Prices of Capital Goods	C	System (IV)	$C\theta$
Production Allotments in Capital Goods Production	$(c + d)C\theta$	System (V)	$(c + d)C\theta$
Profits in Capital Goods Production	$C\theta$		
Quantities of Goods Borrowed and Lent	$G\theta$	System (VI)	$C\theta$
Capitalization Rates ²⁷	1	System (VII)	$C\theta$
Interest Rates ²⁷	$G - (C + c)$		
Discount Rates	θ		
Total	$C\theta(c + d + 3)$ $+ G(\theta + 1)$ $+ \theta + 1 - c$	Total	$C\theta(c + d + 3)$ $+ G(\theta + 1) + C$

No equilibrium on the capital and loan markets without money! Then, far from being inconsistent with static equilibrium, money is in fact essential to it. Let us follow where leads the argument of the writers who claim that money is itself inconsistent with static conditions,

²⁶ Keynes's suggested equation of "the" interest rate with the *highest* commodity rate might also be utilized in this connection (*op. cit.*, pp. 222 f.).

²⁷ Equations (6) of the present essay are used here to reduce the number of these unknowns, rather than to increase the number of equations.

equilibrium or otherwise.²⁸ Money is inconsistent with static equilibrium; static equilibrium is nonexistent without money. Equilibrium then becomes inconsistent with equilibrium, and economic theory assumes the general character of Humpty Dumpty after the fall. If logical consistency in economic reasoning, the testing of which is the major function of general-equilibrium theory,²⁹ is inconsistent with itself, where is a consistent economic reasoner, if such there be, to come in out of this night wherein all cows look black? There is an obvious answer—by explicit introduction of money into the system of capital and lending. The theory of money should not only be self-consistent, but it should so modify barter-capitalization, credit, and discount theory as to bring them into equilibrium and close the gap in the static barter system.

CAPITALIZATION IN A MONETARY ECONOMY

In a monetary economy, at least one commodity is held in cash balances by all θ economic subjects by reason of its superior liquidity.³⁰ When an individual sells a good—let us consider specifically a capital good, C —his receipts will normally take the form of money. On this money he may earn $P_C i_m$, or simply Pi , by lending. He may also purchase annuities worth Pi per period in perpetuity, abstracting from brokerage charges, allowances for uncertainty, etc. He has a triple

²⁸ The view that the use of money becomes rational only in a "dynamic" economy may be found in Hicks's "Gleichgewicht und Konjunktur," *Zeitschrift für Nationalökonomie*, Vol. 2, 1931, especially the section entitled "Gleichgewicht und Geld," and in Dr. P. N. Rosenstein-Rodan's "Co-ordination of the General Theories of Money and Price," *Economica*, Vol. 3 (N.S.), August, 1936, pp. 257–280. These writers' concept of the static economy is more restrictive than that used here. To them, static conditions involve anticipations that all prices will remain constant. I place no such condition on the static (as distinguished from the stationary) economy. The extent of disagreement is exaggerated by the difference in terminology.

²⁹ More attention has been paid in recent years to the derivation of "empirically refutable" propositions from general-equilibrium formulations, primarily by way of second-order stability conditions, than to the original function of the general-equilibrium method, which was to test the logical consistency of the partial-equilibrium results from which the more significant empirically refutable propositions were derived directly. This original function has come to be stigmatized in some quarters as "mere" counting of equations and unknowns, and has fallen into positive and quite unmerited disrepute.

³⁰ Definition of the cash-balance use, liquidity, etc., involves all the knotty problems of the definition of money, and will not be attempted here. I have expressed myself on this in Chapter IV of *Monetary Theory and General Equilibrium* (unpublished dissertation, Chicago, 1939); some aspects of the solution are strikingly similar to B. H. Higgins' formulation in "A Diagrammatic Analysis of the Supply of Loan Funds," *ECONOMETRICA*, Vol. 9, July–October, 1941, pp. 231–240.

choice, then, as holder of C . He may obtain Pi of income by the selling-lending process first outlined, Pi by the process of sale plus purchase of annuity, or $P\lambda$ by selling the *services* of the capital good. In an equilibrium position, choice between these alternatives will be indifferent at the margin.³¹ We therefore must have i equal to λ ; since i is an objectively determined quantity, we can apply the same reasoning which was used to obtain (6):

$$(8) \quad i = \lambda_1 = \lambda_2 = \dots = \lambda_\theta \quad (\theta \text{ equations}).$$

Given equations (8), the θ capitalization rates λ_i are completely determined, and equal to the money rate of interest on loans. Capitalization theory is determinate in the monetary economy, provided that the money rate of interest itself forms part of a determinate theory of credit. There has been no need to utilize investment, production, or turnover "periods" to bring about this determinacy.

This analysis is predicated on the existence of a single money rate of interest. If we consider a situation in which there is a multiplicity of monetary commodities, and further admit that particular commodities may be simultaneously money to one individual and not-money to another, a single rate of interest on money cannot be assumed in advance.

To secure this necessary result, a special condition must be imposed on the money economy. This condition is almost always fulfilled in practice. It may be stated quite simply: There must exist a single loan market, with a single interest rate, for all commodities which are money to any economic subject, which may be lent as they stand (without prior conversion into other monetary forms), and for which a loan market exists. All these monetary commodities must be mutually substitutable, with infinite elasticity of substitution, for borrowing and lending purposes. If this condition is not satisfied, there may exist a plurality of money interest rates, which would rule out any single equilibrium position on the capital and loan markets of the economy.

CREDIT IN A MONETARY ECONOMY

We have found the barter theory of credit to satisfy the necessary conditions for equilibrium, except for an irritating overdeterminacy which arises when elimination of arbitrage between capital and loan

³¹ Varying anticipations of the relative future prices of C , C' , $\dots$, E (the annuity), etc., are formally taken into consideration in the final utility functions of System (I). Individuals' choices as between assets depend in part upon anticipated price changes. Individuals adjust their holdings so that more is held of assets expected to appreciate, less of assets expected to depreciate, than would be the case under stationary assumptions. Indifference at the margin is, however, maintained.

markets is attempted. [See equations (6).] The equilibrium conditions established previously are not disturbed formally by the introduction of money. There is merely a new commodity introduced among those borrowed and lent. (The introduction of money need not eliminate borrowing and lending *in natura*. Neither does it require that all interest rates equal the money rate except in the stationary case.) The overdeterminacy previously existent is not, however, resolved. The $(C+c)$ interest rates on the loans of all types of capital goods and their services still require special attention. The introduction of special equations such as (6), providing equalization between them as a condition for equilibrium, makes the theory of credit overdeterminate.

In order to eliminate this overdeterminacy, a further condition must be imposed upon the monetary economy. Not only must all types of money bear a uniform rate of interest, but all types of capital goods and their services must bear the same rate, if lent or borrowed *in natura*. This condition involves substitution of i_m (or simply i) for $i_c', \dots, i_c, i_c', \dots$, wherever they appear in any of the equation systems.³² This procedure eliminates the portion of (6) that is not included in (8), and thereby renders determinate the theory of credit. It does not otherwise change the number of equations or unknowns.

DISCOUNT IN A MONETARY ECONOMY

Working capital is characteristically held either in inventory or in the most liquid form possible. In the former case, it is not ordinarily lent at interest. In the latter case, the maximum of liquidity involved should be ascertainable with reference both to the present holder and to the prospective seller of the future commodity or obligation. This second portion of the working capital, which is indeed the only relevant

³² The same formal condition might have been imposed on the barter economy. The economic problem would, however, remain: to what loan interest rate under barter would equating the capital-goods and capital-services rates have the same justification as equating them to the money rate in a monetary economy?

A further objection: does this condition not decrease the number of unknowns to the same extent as it replaces equations by identities? If this is true, the overdeterminacy remains. I disagree with this objection, and envisage a solution reached as follows. First, the entire system would be solved, with equations (6) omitted, and with $i, i_c, \dots, i_c, \dots$ treated as separate unknowns. If in the solution, the equilibrium values for i and for some subgroup of capital goods and services were unequal, the inequality would be interpreted as signifying the non-existence, for some purely technical or noneconomic reason, of an *in natura* loan market for these goods and services. A second trial solution would be obtained, with the number of goods and services lent reduced by the number eliminated in the first trial. The process could be repeated (conceptually, at least) until the number of capital goods and services lent is reduced to the number actually dealt in on loan markets otherwise than through the monetary intermediary.

portion for our present purpose, is part of the cash balance. In a monetary economy, it is kept in monetary form. In any economy, the discount rate paid by or to any individual must be related to the interest rate on his working capital. For the monetary case, when only the second portion of the working capital is relevant, each discount rate is related to the interest rate on money, which we have postulated as single-valued. One equation of type (7) therefore applies, when the economy is a monetary one, to each individual. Furthermore, the equations are the same for all individuals. There is only one discount rate, as there is only one monetary interest rate. The indeterminacy of the theory of discount is removed, since equations (7) are equal in number to the discount rates to be determined.

A FINAL TABLE

A summary of the equations and unknowns of the monetary theory of capitalization, credit, and discount is presented as Table III. One monetary commodity has been added to the G nonmonetary ones dealt with previously. (No additional commodities, or a plurality of ad-

TABLE III
EQUILIBRIUM OF THE MONETARY THEORY OF CAPITALIZATION,
CREDIT, AND DISCOUNT

Unknowns	Number	Equations	Number
Quantities of Capital Goods Held	$C\theta$	System (I)	$(G+1)\theta$
Quantities of Capital Goods Produced	$C\theta$	System (II)	$C+G+1$
Prices of Capital Goods	C	System (IV)	$C\theta$
Production Allotments in Capital Goods Production	$(c+d)C\theta$	System (V)	$(c+d)C\theta$
Profits in Capital Goods Production	$C\theta$	System (VI)	$C\theta$
Quantities Borrowed and Lent	$(G+1)\theta$	System (VII)	$C\theta$
Interest Rates	$G+1$	Equations (7)	θ
Capitalization Rates	θ	Equations (8)	θ
Discount Rates	θ		
Total:	$C\theta(c+d+3)$ $+(G+1)(\theta+1)$ $+2\theta+C$	Total:	$C\theta(c+d+3)$ $+(G+1)(\theta+1)$ $+2\theta+C$

ditional commodities, might have been introduced equally well.) The number of equations and unknowns is equal. The necessary condition for the existence of an equilibrium position is satisfied.³³ Subsequent sections will deal with the problem of its stability.

³³ The problem of sufficient conditions is not considered in this study.

CHARACTER OF THE EQUILIBRIUM

The so-called microeconomic techniques of the Lausanne School, with their multiplicity of variables and equations, are generally considered less adapted to the testing of the stable or unstable character of an equilibrium position than to providing a necessary condition for its existence. The former problem yields more readily to macroeconomic techniques involving fewer variables and fewer equations, though it should be noted that the transition involves much more than a reduction in the numbers of equations and unknowns, and also that the reduction may itself conceal problems inherent in the nonhomogeneity of the reduced number of variables. In the field of monetary capital theory, the most fruitful macroeconomic system devised thus far is that of Keynes's *General Theory of Employment, Interest and Money*. While not originally presented in general-equilibrium form, the argument has been cast in general-equilibrium terms by several writers. The best-known such formulation is probably that of Professor Oscar Lange, which has received the approval of Keynes himself.³⁴ Rather than repeat Lange's formulation, I propose to pass on to two further propositions:

1. Even in a monetary economy, the theory of interest generally yields only an *unstable* equilibrium.³⁵ The instability arises when artificial or accidental departures from any equilibrium rate set in motion forces which tend to perpetuate the departures.

2. In the movement of interest and capitalization rates to their equilibrium equality, it is the latter rates which do the greater part of the moving, with the former rates remaining relatively fixed. This is institutionally correct, rather than necessarily true.

Underlying these propositions are two more, equally important in the Keynesian system:

3. Gross monetary saving and investment, considered as flows over time, may be treated as equal in a closed economy, such as we are considering here.

³⁴ "The Rate of Interest and the Optimum Propensity to Consume," *Economica*, Vol. 5 (N.S.), February, 1938, pp. 12-32. Keynes's statement that Lange "follows very closely and accurately my line of thought" occurs in a note to page 318 of his reply to Professor D. H. Robertson, "Mr. Keynes and 'Finance'," *Economic Journal*, Vol. 48, June, 1938, pp. 314-322.

³⁵ As criteria of stability I use two, whose simplest expression is in literary form. The first is a conventional one, perhaps most elegantly expressed by Hicks: "excess demand" should be positive for all prices below equilibrium and negative for all prices above. (As will become apparent from subsequent material "*ex post* minus *ex ante* saving" should be substituted for "excess demand," and interest rate for price, in the definition.) The other test is that any shifts in demand (*ex post* saving) or supply (*ex ante* saving) schedules brought about by a disequilibrium price (interest rate) shall not so alter the equilibrium position as to decrease the magnitude of the disequilibrium.

4. When a monetary authority exists, which can and does expand and contract the monetary circulation in response to the creation and extinction of credit, the monetary income of the community is negatively associated with the rate of interest for a given schedule of marginal efficiency of capital. Since the flow of monetary saving and investment varies in the same direction as money income, it too is associated negatively with the interest rate under the same assumption of a given marginal-efficiency schedule. On the other hand, the positive or direct association of the marginal-efficiency schedule and the level of income may be expected to render the last-named relation, namely that between saving and investment on the one hand and the interest rate on the other, positive on a curve whose slope represents a *total* derivative, while it remains negative on a curve whose slope represents a *partial* derivative of the variables in question. (The possibility of the partial derivative being positive as well will receive separate treatment.)³⁶

Bearing in mind as preliminaries propositions 3 and 4 we may proceed to propositions 1 and 2, which summarize the peculiar characteristics of interest-rate equilibrium. A graphic analysis appears preferable to literary and algebraic alternatives.

The first situation to be illustrated is an equilibrium one. In Figure 1, the curve (*S & I*) represents the schedule of actual money saving and investment corresponding to various interest rates and the resultant money income levels.³⁷ (Marginal time and liquidity preferences are considered given and constant.) The curve S_1' represents the schedule of the amounts of saving which individuals and firms intend to perform *ex ante* at different rates of interest and at an income level corresponding to I_1 .³⁸ The upward slope of S_1' indicates the inducement to save offered

³⁶ If considerations of necessity or policy prevent the circulating medium from varying in either direction with changes in the interest rate, certain of the effects considered above are precluded. Money income, money saving, and money investment remain unaffected by interest-rate changes. The total and partial derivatives mentioned are identical. (Such a monetary policy may be identified with "neutral money," in either of two senses; monetary circulation or gross national income or both are held constant.)

³⁷ This diagrammatic analysis owes a great deal to three earlier presentations, one of which antedates the *General Theory* itself. Compare D. H. Robertson, "Industrial Fluctuations and the Natural Rate of Interest," *Economic Journal*, Vol. 44, December, 1934, pp. 650-656, subsequently reprinted in *Essays in Monetary Theory*, London, 1941; J. R. Hicks, "Mr. Keynes and the 'Classics'; A Suggested Interpretation," *ECONOMETRICA*, Vol. 5, April, 1937, pp. 147-159; and A. P. Lerner, "Alternative Formulations of the Theory of Interest," *Economic Journal*, Vol. 48, June, 1938, pp. 211-230.

³⁸ It is now generally accepted in economic literature that "*ex ante* saving" is a schedule of amounts varying with the interest rate rather than a fixed quantity determined independently of the interest rate.

by higher interest rates at a given level; a downward slope has been suggested as feasible, but would not affect the argument unless its algebraic elasticity were less than that of $(S \& I)$. The intersection of the two curves gives the amount of gross money saving and investment at the equilibrium interest rate I_1 . All the variables have equilibrium values: interest rate, income, saving, and investment.

The instability of this equilibrium may be shown on the same diagram. Suppose the equilibrium rate of interest to be at I_1 as shown, but

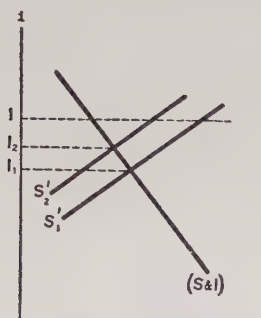


FIGURE 1

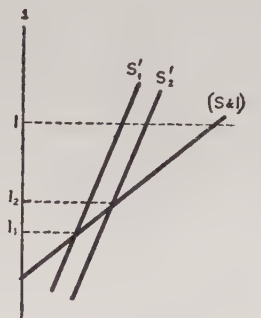


FIGURE 2

suppose the market rate to be I , drawn above I_1 . Then, while the volume of *ex ante* savings out of the income corresponding to I_1 increases, the volume of *ex post* saving, investment, and income will decline. With decreased money income, the curve S_1' shifts to the left, taking up a new position labelled S_2' . The $(S \& I)$ curve, assuming a different income level at every point, remains unchanged. The equilibrium rate of interest rises, to some point above I_1 though not equal to I in the short run. I_2 is not necessarily the new equilibrium rate, as the *ex ante* curve S_2' may not be predicated upon the income level determined by the $(S \& I)$ curve at I_2 . If the market rate continues at I because of frictions or inertia of any sort, the equilibrium rate will continue to rise as income falls, finally approaching I itself as a limit.³⁹ By our second criterion above, we can see that the original equilibrium (at I_1) was unstable, since a disequilibrium rate above I_1 draws the equilibrium rate toward itself. (A disequilibrium rate below I_1 has the same effect in the opposite direction.)

If money is "income-neutral," money income, saving, and investment remain constant *ex hypothesi*.⁴⁰ The $(S \& I)$ curve, which includes

³⁹ This statement is not strictly true regardless of the value of I . See below, p. 57, for a brief discussion of upper and lower limits to the equilibrium rate of interest.

⁴⁰ If it is true that investment plans are made independently of interest rates,

the creations and cancellations of credit, is a vertical line on the diagram. The S' curve of *ex ante* saving is held constant by the stability of money income. The original interest-rate equilibrium at I_1 is stable, and no disequilibrium rate can subsist on a free market. The classical theory of interest is borne out under assumptions of neutral money, just as its incompleteness is shown under a currency system in which the volume of money has a negative interest-rate elasticity. Keynes himself has said as much, in his charge that classical theory assumed the constancy of money income.

No such clear-cut results are yielded by an elastic currency system in which there is a positive relation between the volume of money and the rate of interest. Certain critics of Keynes, notably F. H. Knight, consider this the normal type of elasticity in practice, the Keynesian type being "inverse." Be this as it may, the effect of the assumption of positive elasticity upon the diagrammatic presentation

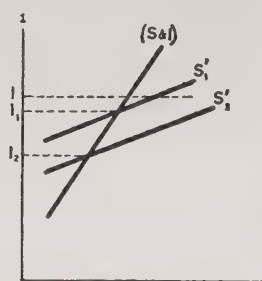


FIGURE 3

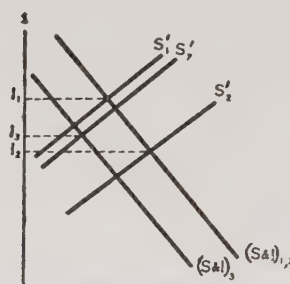


FIGURE 4

here attempted is a positive slope in the $(S \& I)$ curve. Such positively sloping curves are drawn in Figures 2 and 3.

When the $(S \& I)$ curve is upward sloping, the ultimate effect of increased interest rates is considered to be an increase in money income. In such a case, the S' curve tends to move to the right rather than to the left as in Figure 1, when the interest rate I rises above the equilibrium rate I_1 . Figure 2 indicates, however, that even with this assumption the equilibrium at I_1 is unstable under the second aspect of our definition. Only in the case shown in Figure 3, where the elasticity of $(S \& I)$ is not only positive but lower than the elasticities of the family of S' curves, does the maintenance of a disequilibrium interest rate above the equilibrium one tend to lower the latter rather than raise it.

as is suggested by Henderson, Hitch, and Hall in *Oxford Economic Papers*, No. 1, 1938, and by the statistical results of Professor J. Tinbergen's various econometric studies of representative cycles, such indifference may be expected to have the same effect as monetary neutrality.

Only in this last case is a positive interest-rate elasticity of the money and money-income supplies sufficient to render the interest-rate equilibrium a stable one.

This last case, moreover, involves an anomalous situation. According to Figure 3, if individuals intend to save according to a schedule S_1' at the current level of income, a rise of the market interest rate will increase the schedule of intended saving to S_2' by raising the income level. At rates above I_1 , however, the saving actually performed, as read from the $(S \& I)$ schedule, is not only less than the amount intended at these rates out of the higher income but less even than the amount intended out of the lower original income. Despite the volume of abortive saving or hoarding implied by these diagrams, income continues to increase! The pattern of Figure 3 may be discarded as unrealistic, leading to the conclusion that elastic currency renders interest equilibrium unstable, whatever may be the sign of the elasticity.

The interest-rate equilibrium is unstable, however, only within limits. The equilibrium rate need not go to zero or to infinity, whatever the vagaries of market rates. After certain points, changes in income levels need no longer cause S' curves to shift. At the upper limit, further increases in incomes may become fully inflationary and be absorbed completely in consumption. At the lower limit, further decreases may become fully deflationary, with the same effect. In either case, income changes may be offset and finally cancelled by changes in the proportion of income saved as, in Keynesian terms, the marginal propensity to consume approaches unity.⁴¹

This modification is illustrated by Figure 4. It is supposed that, with the equilibrium interest rate at I_1 , a money creation is in progress. There is sufficiently widespread unemployment so that we may postulate a first stage, before bottlenecks cause price increases and before such increases are even anticipated for the near future. In this stage, the effects of the money creation are confined to increased employment, output, and income. The S' curve moves to the right on the diagram, say to S_2' . When and if the effects of price changes are felt, falling liquidity preferences tend to cause the movement of S' to reverse itself somewhat. Suppose the shift to be back to S_3' . The falling liquidity preferences also cause the $(S \& I)$ curve to move to the left, the eventual resting place being the schedule labelled $(S \& I)_3$. These movements of S' and $(S \& I)$ in opposite directions may be expected to lead to a rise in

⁴¹ For a marginal propensity equal to or greater than unity, the foregoing demonstrations of interest-rate instability do not apply, as the movements of the S' curves either do not occur (marginal propensity equal to unity) or occur in the opposite direction to that assumed (marginal propensity greater than unity).

the equilibrium interest rate, possibly even a return to its original level. Suppose that the equilibrium rate does in fact return to I_1 , according to the conclusions of nonmonetary interest theory. Some lower rate, however, such as I_2 , may continue in effect, albeit as a disequilibrium rate, perhaps by reason of conscious decision on the part of monetary authorities. The mechanism illustrated by Figures 1 and 2 then goes into effect. The disequilibrium rate, being below the equilibrium one, tends to draw the equilibrium rate downwards, for the rate at period 3 is neither more nor less stable than was the original rate at period 1. Equilibrium and disequilibrium rates may finally meet well below I_1 , perhaps at I_3 . If so, the effect of increased monetary circulation may be a decrease in the interest rate, equilibrium as well as disequilibrium, long-run as well as short-run. In the Keynesian system, the new equilibrium is associated with higher income, investment, and employment. The important connection with the theme of this essay arises from the fact that in addition to its position as requisite for the determination of any equilibrium interest-rate position whatever, money and monetary changes can alter this position definitively, within wide limits.

RETURN TO MICROECONOMICS

The treatment immediately foregoing has tacitly supposed the shape and position of the curves, S' on the one hand and $(S \& I)$ on the other, as determined by the quantities of money and money income and the attractiveness of undifferentiated "investment" in unidentified capital instruments and titles thereto. In this agglomeration of earning assets it follows the Keynesian model, for in addition to the quantity of money, Keynes's exposition in the *General Theory* considers the rate of interest as determined by liquidity preferences.⁴² These are considered as preferences for money over "earning assets" lumped together. On such a definition as this, increased liquidity preferences (resulting, e.g., from unfavorable anticipations) should result in decreased demand for these assets, higher rates of return over their cost, and higher interest rates.

The macroeconomic agglomeration of earning assets has the disadvantage of neglecting other important liquidity preferences in the economy, particularly the liquidity preferences for certain types of earning assets over others (in particular, for bonds over stocks).⁴³ These

⁴² In Lange's general-equilibrium formulation (*op. cit.*, Sec. 3 and Fig. 1) it is the quantity of cash balances (M) which is made a "liquidity-function" of the interest rate and the level of income. In his terminology, $M = L(i, Y)$.

⁴³ The so-called "asset theory" of money, associated with the name of Professor Jacob Marschak, is essentially an attempt to combine microeconomic analysis with a Keynesian point of view. Marschak's distinctions between payments for

are important in the explanation of concrete capital-market phenomena. Specifically, when liquidity preferences for money over assets increase, preferences for certain other assets (notably short-term government paper) are likely to increase as well. Their rates of return are likely to fall. Since such assets as short-term government paper, the very assets "complementary with money against other assets," are those whose rates of return correspond most closely to pure economic interest, it is possible to claim that increased liquidity preferences lead normally to decline of "the" interest rate, a result quite contrary to the implications of Keynesian thought.

The question is clear: to what extent does the characteristically Keynesian case hold when the critic makes a first step back toward microeconomics by segregating earning assets into broad subclasses?

Three methods of reconciling the frequently observed lowering of bond rates following increased liquidity preferences with the essentially Keynesian interest theory supported here may be suggested.

1. The "rate of interest" which Keynes considers "the inverse proportion between a sum of money and what can be obtained for parting with control over the money in exchange for a debt,"⁴⁴ should be interpreted as the difference between the rate of return on high-grade bonds and that on other assets, which may be considered collectively as "debts," notwithstanding the equity nature of many of them.

2. For at least certain assets (whose major attraction is high productivity), account should be taken of an effect stressed by de Scitovszky, the positive interrelation between the marginal efficiency of capital and its rate of accumulation, as distinguished from the quantity in existence.⁴⁵ In good times, the marginal efficiency of such types of assets rises as compared with that of other types, and in bad times it falls. Indeed, where the principal attraction of the asset is great safety, a negative interrelation is more probably in evidence, as the premium for safety decreases if equally safe assets are burgeoning.

the "safety," "plasticity," and "lucrativity" of assets are highly relevant to the present discussion. See particularly Makower and Marschak, "Assets, Prices, and Monetary Theory," *Economica*, Vol. 5 (N.S.), August, 1938, pp. 261-288, esp. p. 287. T. de Scitovszky likewise, in his "Study of Interest and Capital," *ibid.*, Vol. 7 (N.S.), August, 1940, pp. 293-317, esp. p. 300, speaks of particular types of securities as complementary with money against other assets.

⁴⁴ Keynes, *General Theory*, p. 167. In a note to this passage, he continues: "We can draw the line between 'money' and 'debts' at whatever point is most convenient for handling a particular problem. . . . It is often convenient in practice to include in *money* time-deposits with banks and, occasionally, even such instruments as (e.g.) Treasury bills."

⁴⁵ De Scitovszky, *op. cit.*, pp. 314-317. (Credit for the suggestion is given to Professor A. P. Lerner.)

3. Liquidity "preferences" or "functions" should be redefined so as to consider explicitly their movements in directions opposite to capitalization rates or marginal efficiencies. Keynes does not consider such movements, preferring to treat liquidity preferences as connected only tenuously if at all with marginal efficiencies. This is passing strange, since he emphasizes movements of liquidity preferences opposite in direction from those of money income.

When we consider liquidity preferences and marginal efficiency as closely interrelated, any rise in liquidity preferences may be looked upon as having two major accompaniments. In its association with falling capitalization rates, it will tend to lower the equilibrium interest rate for any given level of money income, to move to the left the (*S & I*) curves of the diagrams. In its association with decreases in money income, it will tend to raise the market rate of interest above the equilibrium rate for the *previous* income level. Its net effect on market rates is not invariably in either direction, being the sum of two opposing forces.

Liquidity preferences tend in practice to rise markedly during two types of situation, *money panic* and *business depression*, or perhaps rather in the acute and chronic phases of the same economic malady. In the former acute case, the disequilibrating effect of increased liquidity preferences on money income dominates, and interest rates tend to rise. In the latter chronic one, the effect of increased liquidity preferences on the equilibrium rate for a given income dominates, and interest rates tend to fall.

What, finally, becomes of the strict Keynesian position, when earning assets are thus segregated or "deglomerated"? It survives most fully, with the greatest claim to realism and least need of modification, in a situation of money panic. In situation of prolonged depression or secular stagnation, which it sets itself primarily to explain, the theory loses by its neglect of the interrelations existent between capitalization rates and liquidity preferences. While it requires expansion, perhaps in some of the directions indicated here, none of these expansions will deprive it of its essentially monetary character, its most controversial characteristic.

United States Treasury

